Nuclear Physics
SEM III P II UNIT I

By
Dr Dinesh V Kala
Head, Dept. of Physics
Guru Nanak Khalsa College Matunga, Mumbai-19

Unit 1		Static properties of nuclei
	1.1	All (charge, mass, binding energy, size, shape, angular momentum, magnetic dipole momentum, electric quadrupole momentum, statistics, parity, isospin); Measurement of Nuclear size and estimation of R0 (mirror nuclei and mesonic atom method); Q-value equation; Energy release in fusion and fission reaction.
	1.2	Deuteron Problem and its ground state properties; Estimate the depth and size of (assume) square well potential; Tensor force as an example of non-central force; Nucleon-nucleon scattering-qualitative discussion on results; Spin-orbit strong interaction between nucleon; Double scattering experiment.

Questionnaire I

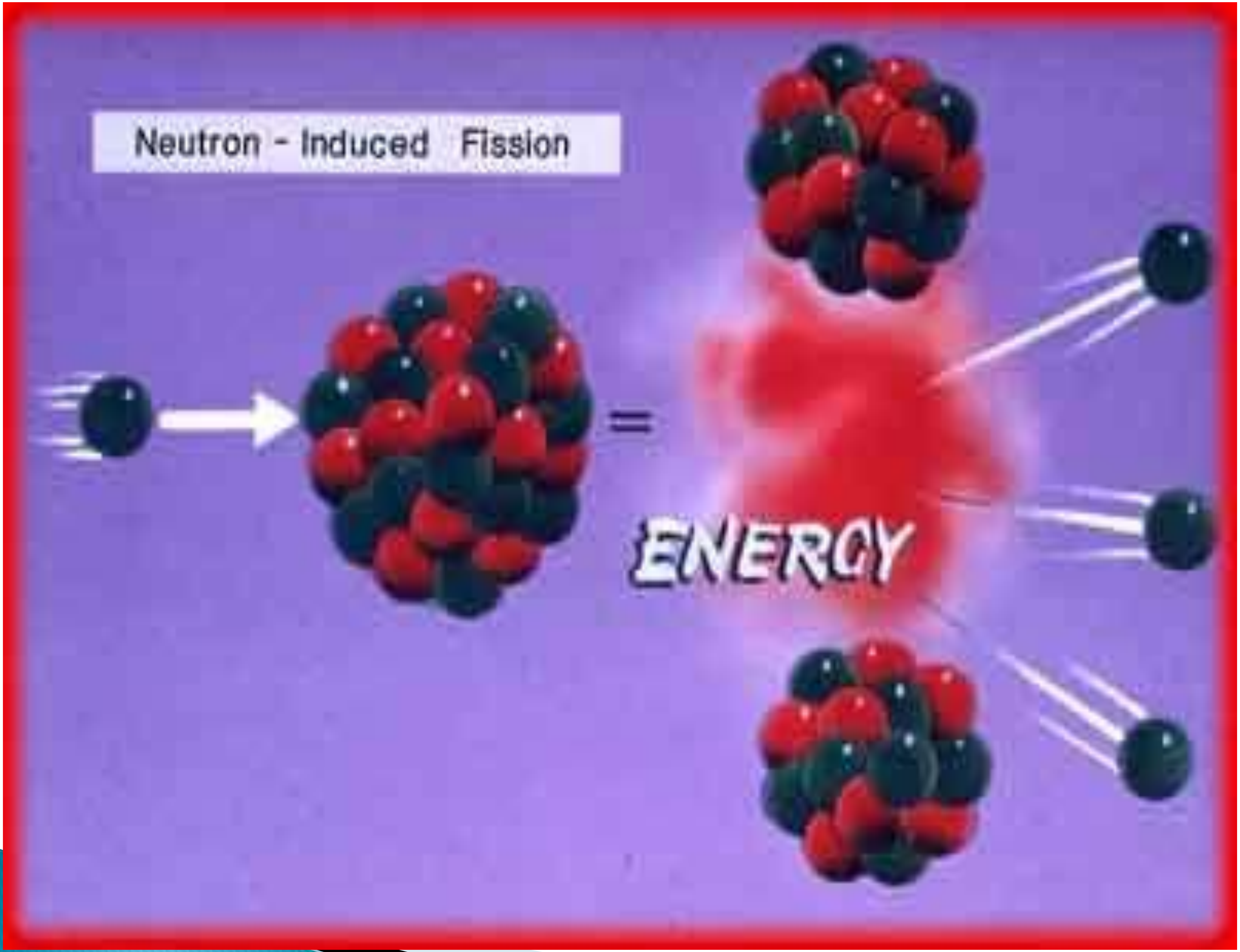
1. Charge of Nucleus
2. Nucleons ? How many is Deuteron ?
3. Nuclear Force ? Range ? 1 femto ?
4. Isotopes
5. Isotones
6. Isobars
7. 1 eV = ? Joules; I amu = ? Kg
8. 1 amu is equivalent to ? MeV

Questionnaire II

1. **Alpha Particle & Its Emission.**
2. **Beta Particle & Its Emission.**
3. **Gamma Particle & Its Emission.**
4. **Mass defect & Binding Energy.**
5. **Different Nuclear Models.**
6. **Heavy Nuclei & Radioactivity.**
7. **Isotopes of Hydrogen.**

Questionnaire III

1. Quantum numbers and relation between them?
2. Find all Q numbers for PQN; n= 3
3. Reason & Result of Rutherford Experiment?
4. Average BE per nucleon ?
5. BE per nucleon for the most stable nuclei ?
6. Reduced Mass ? Its Advantage?
7. Central Force? What is Special about CF?
8. Value of l for ground state?


Neutron - Induced Fission
ENERGY

FISSION

- $^{235}U_{92} + {}^{1}n_{0} = {}^{141}Ba_{56} + {}^{92}Kr_{36} + 3\ {}^{1}n_{0}$
- $236.0525u = 235.8666u$

FISSION

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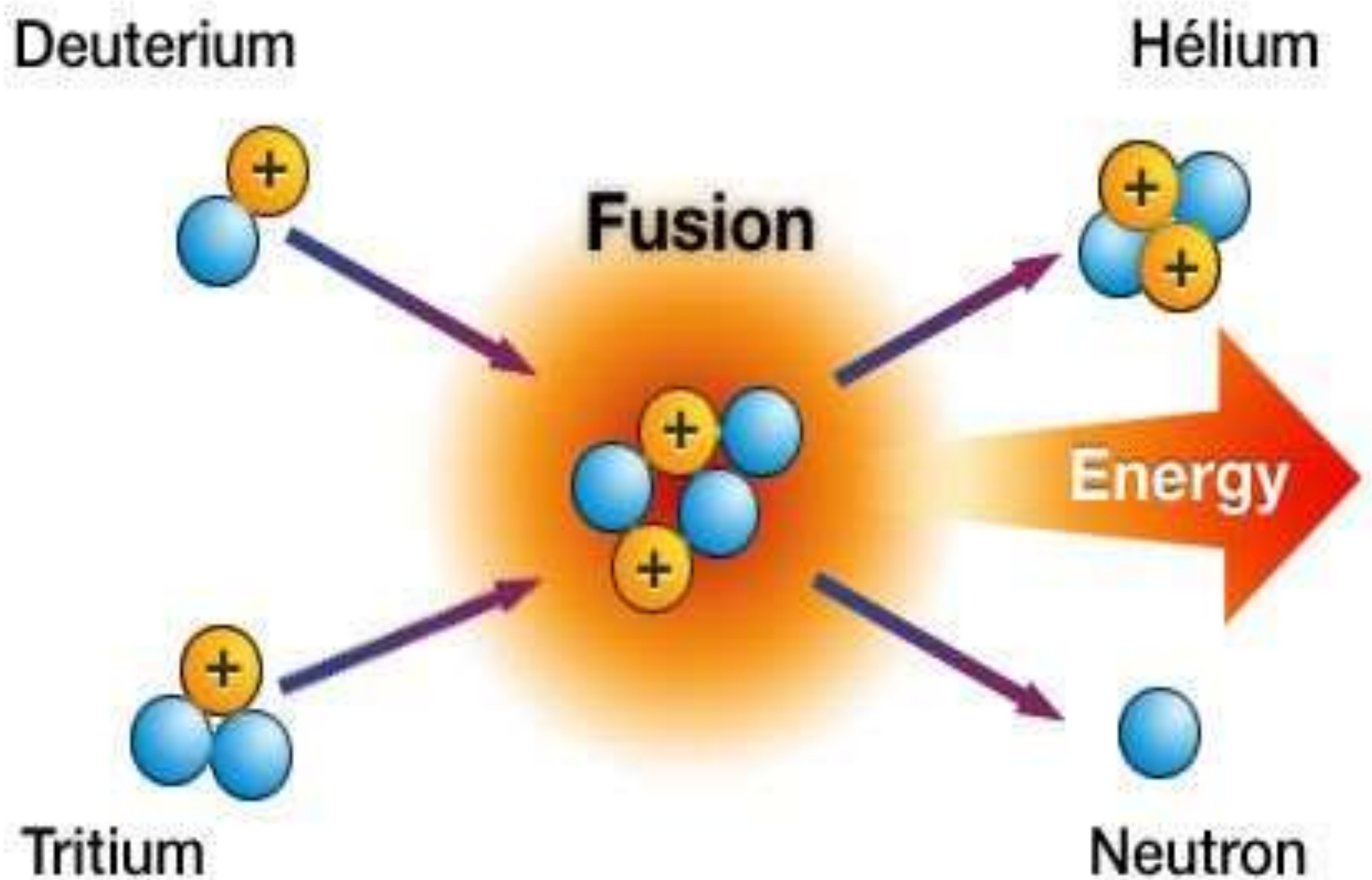
- 236.0525u = 235.8666u
- ▲M=0.1859u
- E = ▲M C^2
- $2.78x10^{-11}$J
- $1.67x10^{10}$KJ/Mole


Deuterium
Hélium
Fusion
Energy
Tritium
Neutron

FUSION

- $^{2}H_{1} + {}^{3}H_{1} = {}^{4}H_{2} + {}^{1}n_{0}$
- 5.029602 u = 5.010171 u

FUSION

- $^{2}H_{1} + {}^{3}H_{1} = {}^{4}H_{2} + {}^{1}n_{0}$
- 5.029602 u = 5.010171 u
- ▲M=0.019431u
- $E = ▲M\ C^{2}$
- $2.91x10^{-12}J$
- $1.75x10^{9}$ KJ/Mole

Fusion & Fission

Fusion is what powers the SUN. Atoms of Tritium and Deuterium (isotopes of hydrogen, Hydrogen-3 and Hydrogen-2, respectively) unite under extreme pressure & temperature to produce a Neutron & a Helium isotope.

Along with this, an enormous amount of energy is released, which is several times the amount produced from fission.

Fusion & Fission

Both fission & fusion are nuclear reactions that produce energy, but the applications are not the same. Fission is the splitting of a heavy, unstable nucleus into two lighter nuclei, & fusion is the process where two light nuclei combine releasing vast amounts of energy.

Fission is used in nuclear power reactors since it can be controlled, while fusion is not utilized to produce power since the reaction is not easily controlled and is expensive to create the conditions for a fusion reaction.

Fusion & Fission

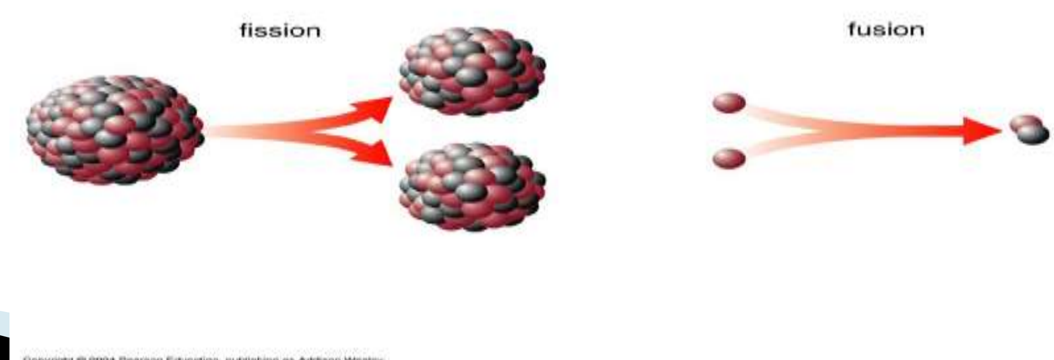
Nuclear fission takes place when a large, somewhat unstable isotope is bombarded by high-speed particles, usually neutrons. These neutrons are accelerated and then slammed into the unstable isotope, causing fission.

During the process, a neutron is accelerated and strikes the target nucleus, which in most nuclear power reactors today is Uranium-235.

This splits the target nucleus & breaks it down into two smaller isotopes (the fission products), three high-speed neutrons and a large amount of energy.

Fusion & Fission

This resulting energy is then used to heat water in nuclear reactors & ultimately produces electricity. The high-speed neutrons that are ejected become projectiles that initiate other fission reaction finally leads to chain reactions.



Fusion & Fission

The word fusion means "a merging of separate elements into a unified whole". Nuclear fusion refers to the "union of atomic nuclei to form heavier nuclei resulting in the release of enormous amounts of energy“.

Fusion takes place when two low-mass isotopes, typically isotopes of hydrogen, unite under conditions of extreme pressure and temperature.

Force between Two Nucleons

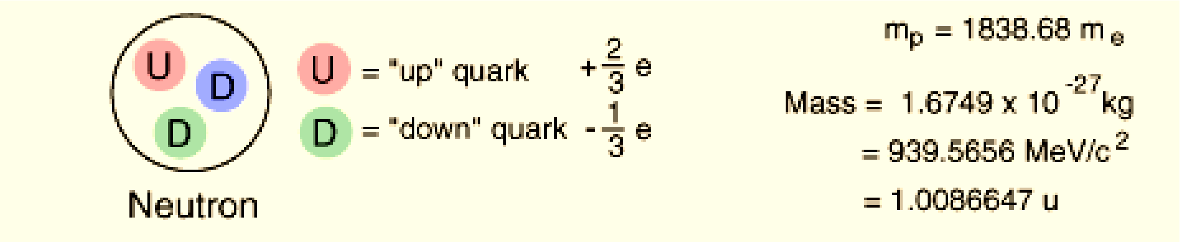
The Deuteron Problem

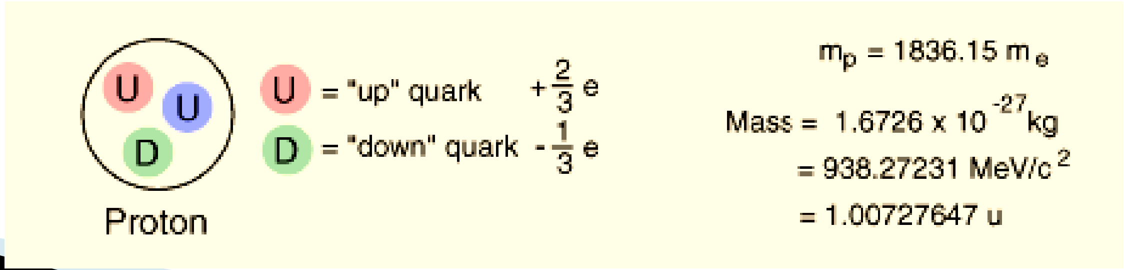
Hydrogen for Atomic Physics
Deuteron for Nuclear Physics

The Deuteron

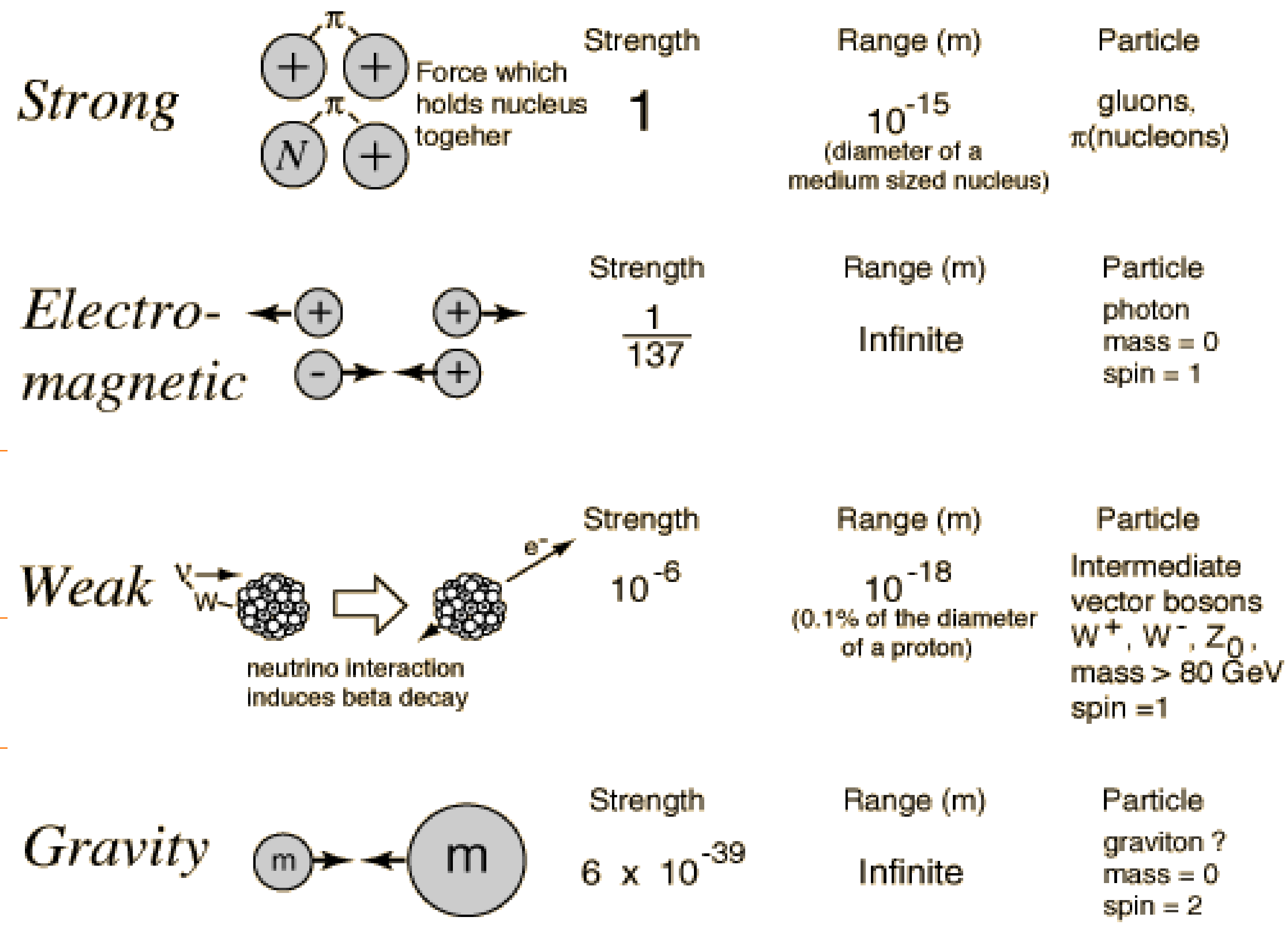
The deuteron, composed of a proton and a neutron, is a stable particle. abundance of 1.5 x 10-4 compared to 0.99985 for ordinary hydrogen.

Constituents	1 proton 1 neutron
Mass	2.014732 *u*
Binding energy	2.224589 ± 0.000002 MeV
Angular momentum	1
Magnetic moment	0.85741 ± 0.00002 μ_N
Electric quadrupole moment	+2.88 x 10^{-3} bar
RMS separation	4.2 fm





Fundamental Forces

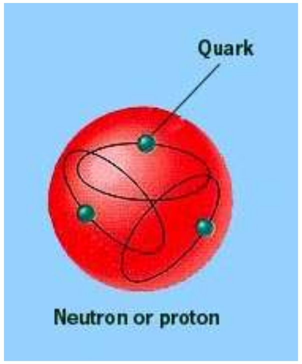


A few properties of nucleon-nucleon force:

1. At short distances it is stronger than the Coulomb force; the nuclear force can overcome the Coulomb repulsion of protons in the nucleus.

2. At long distances, of the order of atomic sizes, the nuclear force is negligibly feeble; the interactions among nuclei in a molecule can be understood based only on the Coulomb force.

3. Some particles are immune from the nuclear force; there is no evidence from atomic structure, for example, that electrons feel the nuclear force at all.



Some other remarkable properties of the nuclear force:

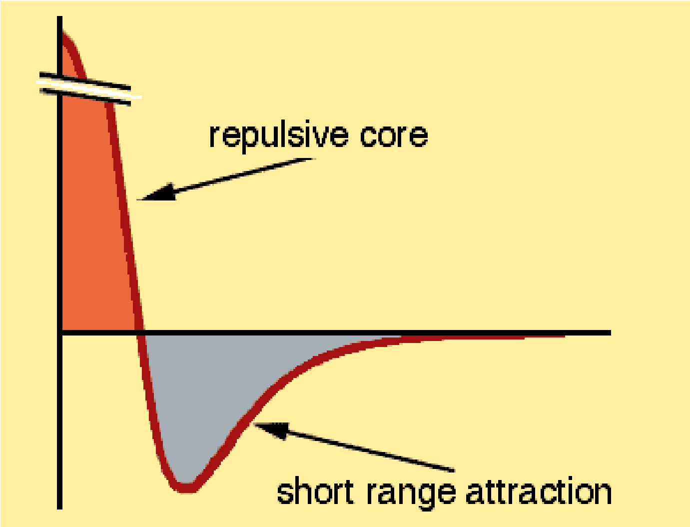
1. The nucleon-nucleon force seems to be nearly independent of whether the nucleons are neutrons or protons. This property is called *charge independence*.

2. Nuclei have same density ($R=R_oA^{1/3}$) and the binding energy per nucleon for nuclei A>40 is constant. These facts imply that *nuclear force saturates*. Nucleons attract each other strongly only if they are in same orbital state.

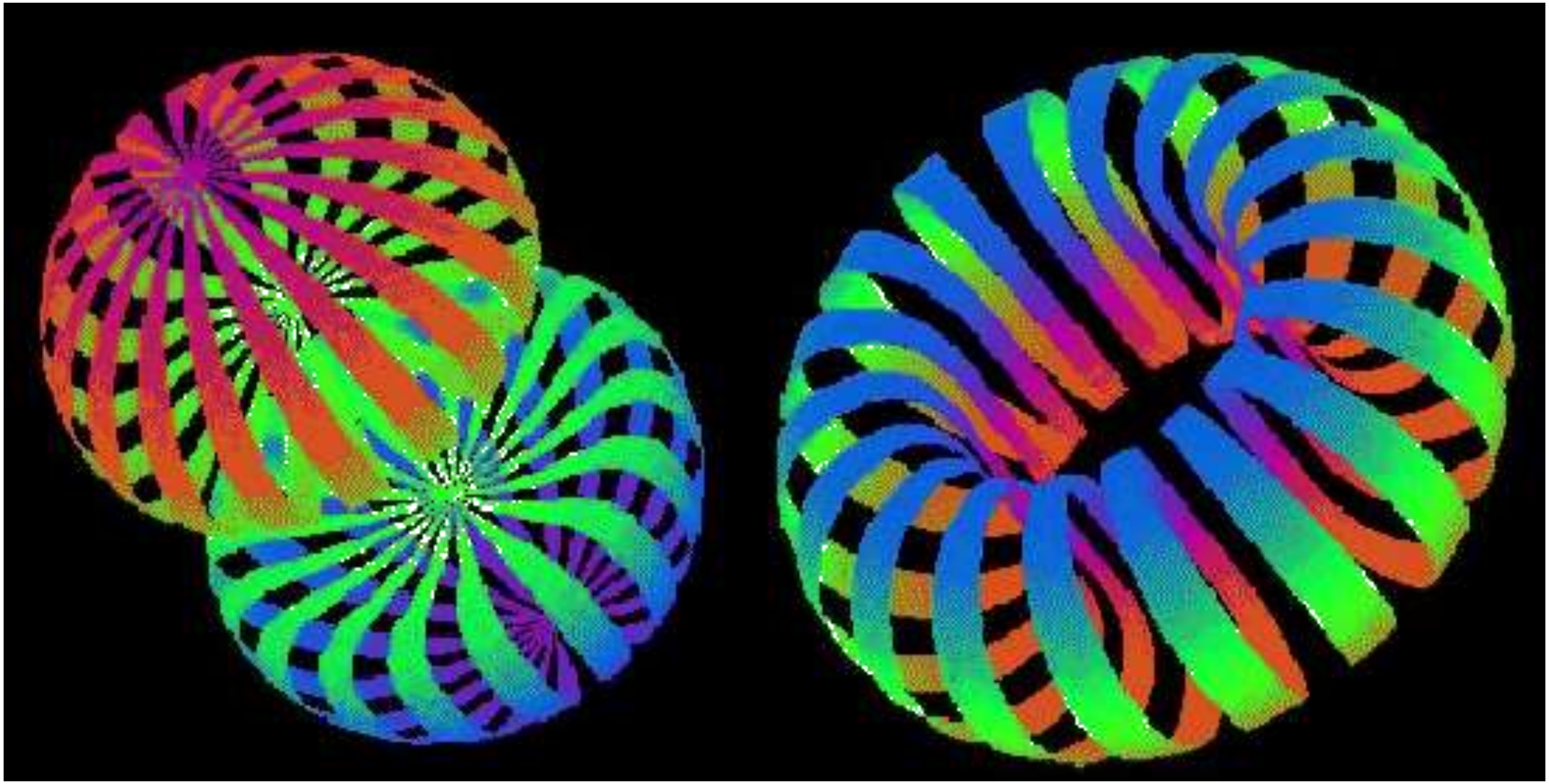
3. The nucleon-nucleon force depends on whether the spins of the nucleons are parallel or anti-parallel.

4. The nucleon-nucleon force includes a repulsive term, which keeps the nucleons at a certain average separation.

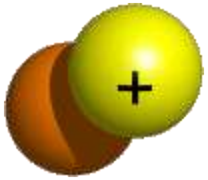
5. The nucleon-nucleon force has a *non-central* or *tensor* component. This part of the force does not conserve orbital angular momentum, which is a constant of the motion under central forces.

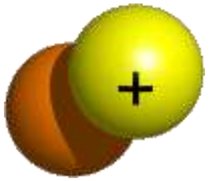


The deuteron



Shapes of the deuteron in the laboratory reference frame.
Stripes show surfaces of equal density for the M_J =1 (left) and M_J =0 (right) magnetic substates of the J =1 ground state.



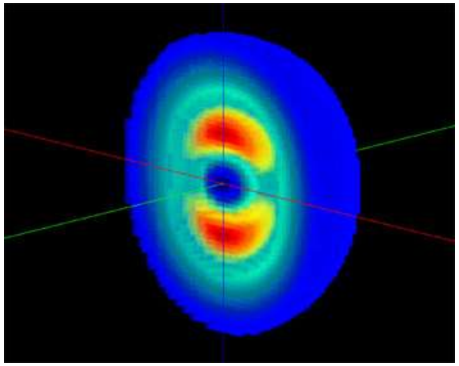


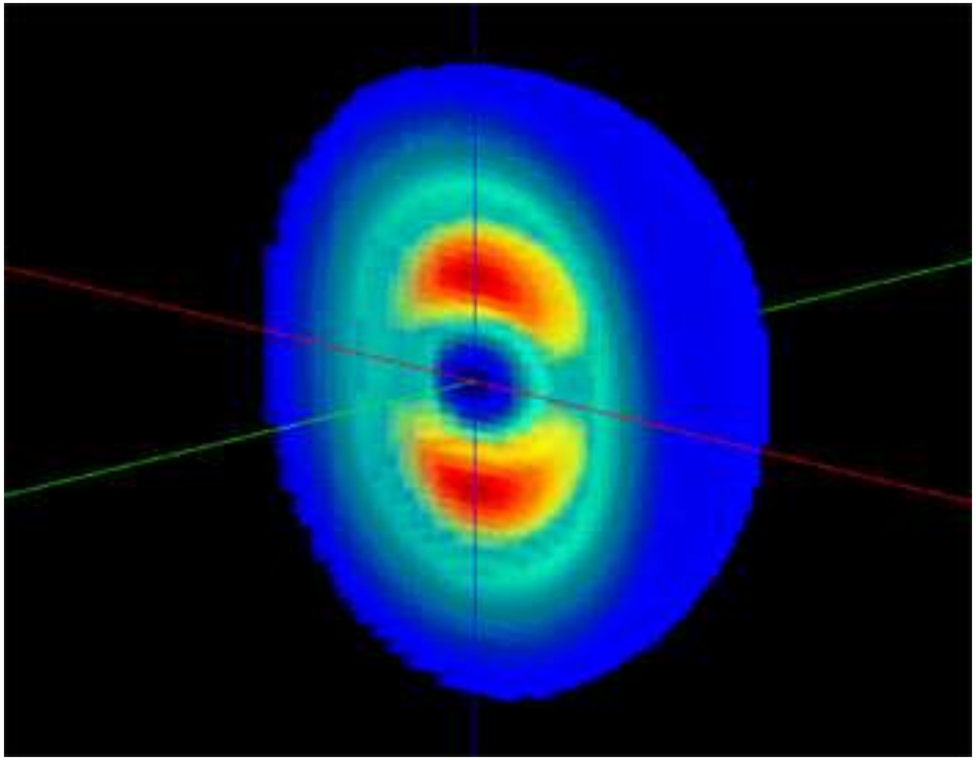
The Deuteron

1. A deuteron (^{2}H nucleus) consists of a *neutron* and a *proton*.
   (A neutral atom of ^{2}H is called *deuterium*.)

2. It is *the simplest bound state* of nucleons and therefore gives us an ideal system for studying the nucleon-nucleon interaction.

3. An interesting feature of the deuteron is that it does not have excited states because it is *a weakly bound system*.





This image shows the intrinsic shape of the deuteron

- The simplest nucleus in nature is that of the hydrogen isotope, deuterium.
- Known as the “deuteron,” the nucleus consists of one proton and one neutron.
- Deuteron is an ideal candidate for understanding the basics of nuclear physics.

4. The Deuteron - Angular momentum

1. In analogy with the ground state of the hydrogen atom, it is reasonable assume that the ground state of the deuteron also has zero orbital angu momentum **L = 0**
2. However the total angular momentum is measured to be **I = 1** (one unit of h/ thus it follows that the proton and neutron spins are parallel. $\mathbf{s_n + s_p = 1/2 + 1/}$ **1**
3. The implication is that two nucleons are not bound together if their spins anti-parallel, and this explains why there are no proton-proton or neutr neutron bound states.
4. The parallel spin state is forbidden by the Pauli Exclusion Principle in the case identical particles.
5. The nuclear force is thus seen to be **spin dependent**.

5. **The parity of deuteron as measured by studies of nuclear disintegrations and reactions is found to be even.**
6. **The sum of the magnetic dipole moments of the proton ($2.79275\mu_N$) and neutron ($-1.91315\mu_N$) do not exactly equal to the magnetic moment ($0.85735\mu_N$) of the deuteron.**
7. **Quadrupole moment of the deuteron is 0.00282×10^{-28} m^2.**
8. **This shows departure from spherical symmetry of a charge distribution.**
9. **+Ve sign shows that distribution is prolate rather than oblate.**

The Deuteron - Binding energy

Binding energy of the deuteron is 2.2 MeV.

If the neutron in the deuteron were to decay to form a proton, electron and antineutrino, the combined mass energies of these particles would be 2(938.27 MeV) + 0.511 MeV = 1877.05 MeV

But the mass of the deuteron is 1875.6 MeV !!

As we have discussed previously, the average binding energy per nucleon is about 7 ~ 8 MeV for typical nuclei. The binding energy of the deuteron, B = 2.224 MeV, is *away too small when compared with typical nuclei*. This means that *the deuteron is very weakly bound*.

Here we want to explore more about this result and study the properties of the deuteron.

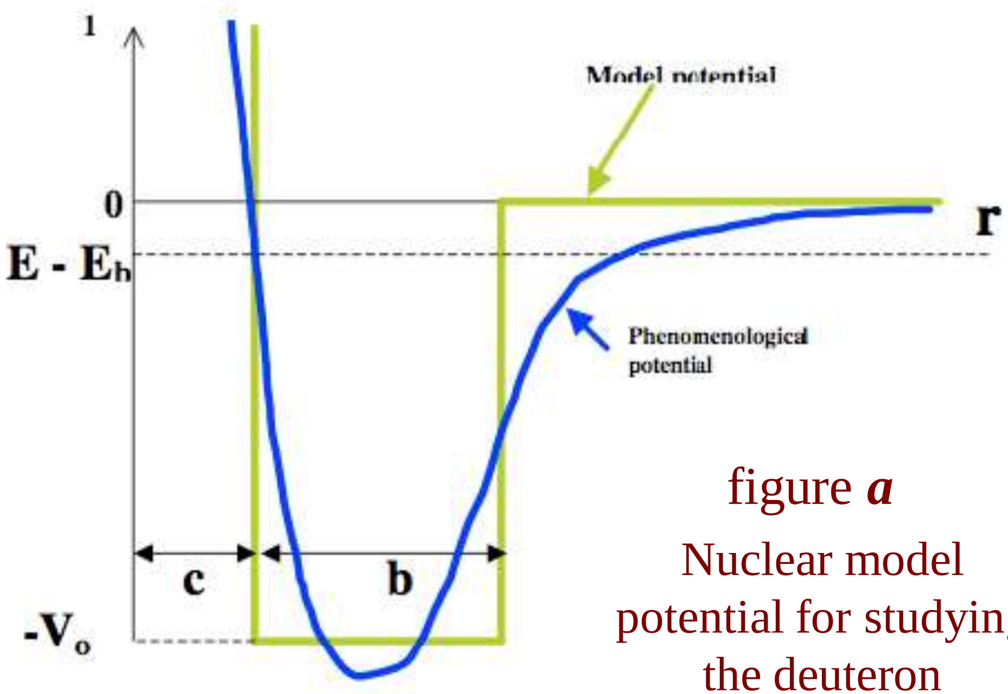
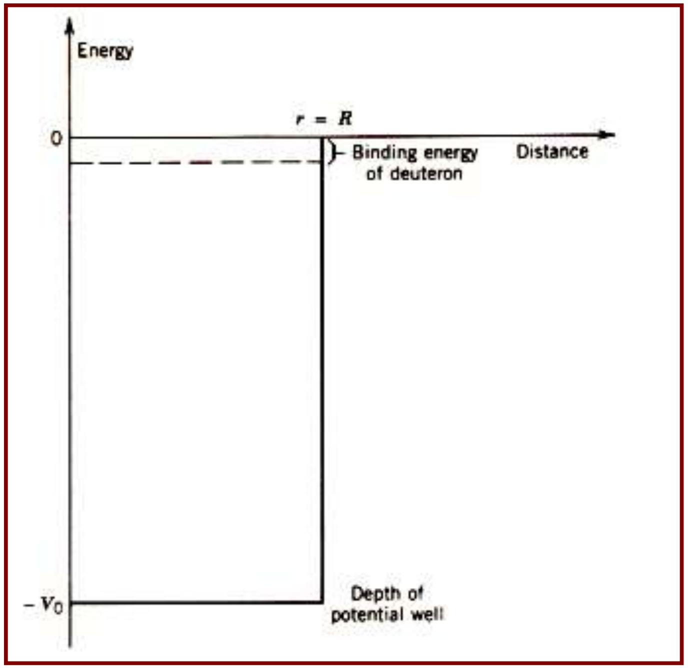


figure ***a***

Nuclear model potential for studying the deuteron

To simplify the analysis of the deuteron, we assume that the nucleon-nucleon potential is *a three-dimensional square well*, as shown in the figure ***a***:

Quantum mechanical description of the weak binding for the deuteron



$$V(r) = -V_0 \quad \text{for } r < R$$
$$= 0 \quad \text{for } r > R \qquad (4)$$

Here *r* represents the separation between the proton and the neutron, so *R* is in effect a measure of the *diameter* of the deuteron.

The dynamical behavior of a nucleon must be described by the Schrödinger's equation:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r}) + V(r)\Psi(r) = E\,\Psi(\vec{r})$$

where *m* is the nucleon mass. (5)

If the potential is not orientationally dependent, a central potential, then the wave function solution can be separated into *radial* and *angular* parts:

$$\Psi(r) = R(r)Y_{lm}(\theta,\varphi) \qquad (6)$$

Substitute $R(r) = u(r)/r$ in to the Schrödinger's equation the function $u(r)$ satisfies the following equation ;

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2}+\left\{V(r)+\frac{l(l+1)\hbar^2}{2mr^2}\right\}u(r)=Eu(r) \qquad (7)$$

The solution $u(r)$ is labeled by two quantum numbers n and l so that:

$$u(r)\rightarrow u_{nl}(r) \qquad (8)$$

The full solution $\Psi(\mathbf{r})$ then can be written as

$$\Psi(\vec{r})=\psi_{nlm}(r,\theta,\varphi)=R_{nl}(r)Y_{lm}(\theta,\varphi) \quad \text{with} \quad R_{nl}(r)=\frac{u_{nl}(r)}{r} \qquad (9)$$

Three quantum numbers to define an eigenstate

- ***n***: the principal quantum number which determines the energy of an eigenstate.
- ***l***: the orbital angular momentum quantum number.
- ***m***: the magnetic quantum number, $-l \leqq m \leqq l$.

The angular part of the solution $Y_{lm}(\theta,\varphi)$ is called the "*spherical harmonic*" of order *l*, *m* and satisfies the following equations:

$$\hat{L}^2 Y_{lm}(\theta,\varphi) = l(l+1)\hbar^2 Y_{lm}(\theta,\varphi) \quad \text{and} \quad \hat{L}_Z Y_{lm}(\theta,\varphi) = m\hbar Y_{lm}(\theta,\varphi) \qquad (10)$$

$$\text{where} \quad \hat{L}^2 \equiv -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right] \quad \text{and} \quad \hat{L}_Z \equiv -i\hbar\frac{\partial}{\partial\varphi} \qquad (11)$$

For the case of a three dimensional square well potential with *zero angular momentum* ($l = 0$), which we use as the model potential for studying *the ground state of the deuteron (spherically symmetric, s-state)*,
the Schrödinger's equation can be simplified into:

$$\begin{cases} -\dfrac{\hbar^2}{2m}\dfrac{d^2u}{dr^2} - V_0 u(r) = Eu(r) & \text{, for } r < R \\[2ex] -\dfrac{\hbar^2}{2m}\dfrac{d^2u}{dr^2} = Eu(r) & \text{, for } r > R \end{cases} \qquad (12)$$

The Schrödinger's equation is $-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} - V_0 u(r) = Eu(r)$ **When *r* < *R*** (13a)

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} = Eu(r)$$ **When *r* > *R*** (13b)

These equations can be rearranged into:

$$\frac{d^2u}{dr^2} + k_1^{\,2}u(r) = 0 \quad \text{with} \quad k_1 \equiv \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$$ **When *r* < *R*** (14a)

$$\frac{d^2u}{dr^2} - k_2^{\,2}u(r) = 0 \quad \text{with} \quad k_2 = \sqrt{\frac{-2mE}{\hbar^2}}$$ **When *r* > *R*** (14b)

General solutions of eqs. (14a) & (14b) are:

$$u(r) = A\sin k_1 r + B\cos k_1 r$$ **When *r* < *R*** (15a)

$$u(r) = Ce^{-k_2 r} + De^{+k_2 r}$$ **When *r* > *R*** (15b)

Following boundary conditions must be imposed:

To keep the wave function finite for $r \to 0$ $\quad \lim_{r\to 0} \psi(r) = \lim_{r\to 0} \frac{u(r)}{r} = 0$ (16)

To keep the wave function finite for $r \to \infty$ $\quad \lim_{r\to\infty} u(r) = 0$ (17)

The coefficient *B* & D must be set to zero.
Therefore the acceptable solution of physical meaning is

$$u(r) = A \sin k_1 r$$ **When *r* < *R*** (18)

$$u(r) = Ce^{-k_2 r}$$ **When *r* > *R*** (19)

Applying the continuity conditions on $u(r)$ and du/dr at $r = R$, we obtain

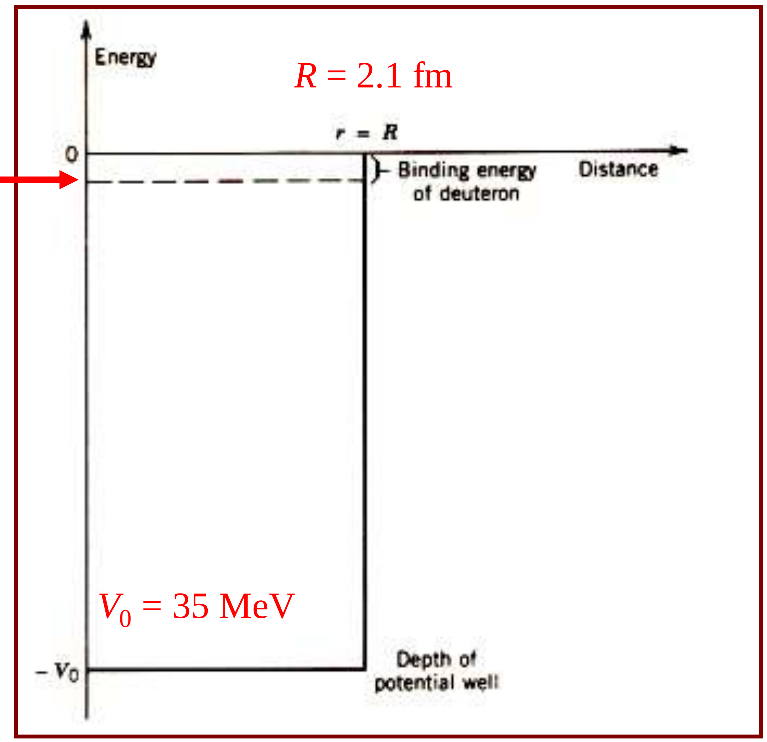
$$k_1 \cot k_1 R = -k_2 \tag{20}$$

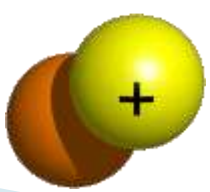
This transcendental equation gives a relationship between V_0 and *R*.

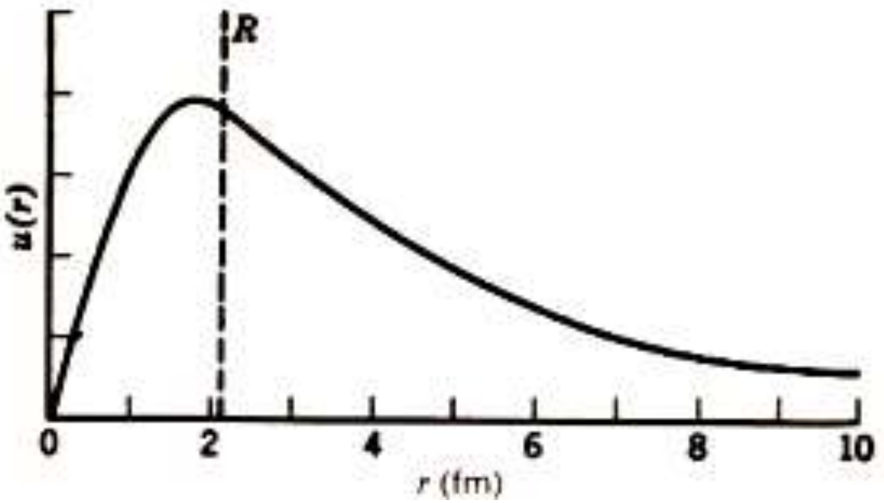
From electron scattering experiments:
the *rms* charge radius of the deuteron is known to be about 2.1 fm.
Taking R = 2.1 fm we may solve from equation (20), the value of the potential depth V_0.
The result is V_0 = 35 MeV.

And also, nucleons in the deuteron spend only 1/3 rd time within the range of nuclear force and therefore, deuteron is loosely bound.

The bound state of the deuteron, at an energy of about -2 MeV, is very close to the top of the well.







- The graph shows that the deuteron wave function for $R = 2.1$ fm.
- The exponential joins smoothly to the sine at $r = R$, so that both $u(r)$ and du/dr are continuous.
- Radial distance where amplitude decreases to 1/e of its maximum amplitude is called **radius of deuteron** ($\approx 2R$) which shows that deuteron is loosely bound.

Spin and parity of the deuteron

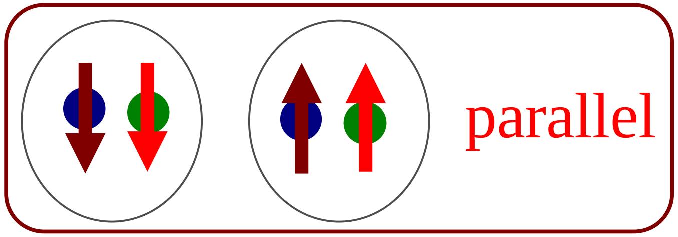
There are four ways to couple $\boldsymbol{s}_n$, $\boldsymbol{s}_p$, and $\boldsymbol{l}$ to get a total $\boldsymbol{I}$ of 1.

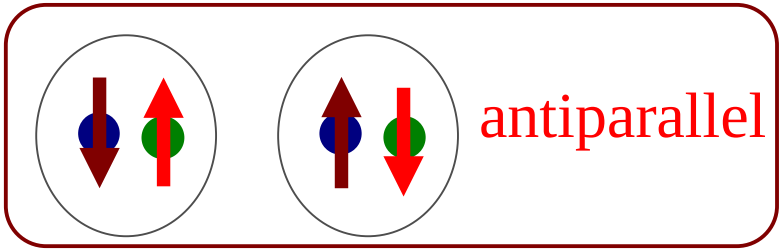
(a) $\boldsymbol{s}_n$ and $\boldsymbol{s}_p$ *parallel* with $\boldsymbol{l} = 0$

(b) $\boldsymbol{s}_n$ and $\boldsymbol{s}_p$ *antiparallel* with $\boldsymbol{l} = 1$

(c) $\boldsymbol{s}_n$ and $\boldsymbol{s}_p$ *parallel* with $\boldsymbol{l} = 1$

(d) $\boldsymbol{s}_n$ and $\boldsymbol{s}_p$ *parallel* with $\boldsymbol{l} = 2$





- Orbital angular momentum $l = 0$ and $l = 2$ give the correct parity determined from experimental observations.

The deuteron is **96%** $l = 0$ (s orbit) and only **4%** $l = 2$ (d orbit).